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Long-range spatial correlations for anisotropic zero-range processes

Christian Maes†§ and Frank Redig†‖

† Instituut voor Theoretische Fysica, Leuven, Belgium
‡ Department Natuurkunde Universiteit Antwerpen, Belgium

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Abstract. The spatial correlations are investigated for a homogeneous system of indistinguishable particles undergoing stochastic anisotropic hopping dynamics on the d-dimensional lattice, $d \geqslant 2$. The interaction is zero range, i.e. the rate at which particles leave a given site only depends on the occupation number at that site. A series expansion around the independent particle system is given for the equal time correlations and is shown to converge for small times t. The formal $t \to \infty$ limiting expansion is analysed termwise from which a quadrupole type decay $(\sim r^{-d})$ is derived for the stationary two-points function $\langle \eta(0)\eta(r)\rangle - \langle \eta(0)\rangle^2$. This phenomenon of self-organized criticality is a direct consequence of the anisotropy causing the system to violate the condition of detailed balance, combined with the conservation law forcing a diffusive decay $(\sim t^{-d/2})$ of the temporal correlations.

1. Introduction

Consider a particle system on the d-dimensional lattice $\mathbb{Z}^d$. The number of particles at each site $x \in \mathbb{Z}^d$ is a non-negative integer $\eta(x)$ and the full particle configuration is completely specified by an element

$$\eta \equiv \{\eta(x);\, x \in \mathbb{Z}^d\} \in \mathbb{N}^{\mathbb{Z}^d}.$$

The motion of the particles is described by nearest-neighbour hopping dynamics in such a way that the rate at which a particle at x moves to a nearest neighbour y, $|x-y| = 1$, depends on the particle configuration only through the number of particles which are at x at that time. The particles are indistinguishable and an independent particle system therefore has rates proportional to the particle number; it is equivalent to a system of independent random walkers. When an interaction is superimposed in the above sense on this independent motion, then we speak about a zero-range process. It was introduced by Spitzer [1] in 1970 and has been greatly studied since then, see the references in [2]. In particular, the hydrodynamics of the zero-range process has been investigated in detail by a number of authors [3–6], and used to test a number of fundamental methods and principles of non-equilibrium statistical mechanics, see e.g. [7].

Here we will be concerned with the microscopic system and ask in what sense a zero-range process—with 'small' interactions—can be close to the independent particle

§ Aangesteld Navorser NFWO, Celestijnenlaan 200D, B-3001 Leuven, Belgium.
‖ Aspirant NFWO, Universiteitsplein 1, B-2610 Wilrijk, Belgium.

system. We must therefore apply a generic perturbation and look for its consequences. We have chosen for the time-honoured practice of expanding around an exactly soluble model, in this case, that of independent random walkers. While we consider this problem to be interesting in itself, our work has been triggered especially by recent investigations on the existence of long-range spatial correlations in stationary states of generic non-equilibrium conservative dynamics. We refer to [8] for a detailed discussion. Similar ideas and arguments as in [8] will be made explicit here for the zero-range process.

The only analysis we know of, investigating this so-called 'self-organized criticality' for zero-range processes, is a heuristic derivation of the fluctuating hydrodynamics by Van Beijeren in [9] for two-dimensional (2D) dynamics in which the particles jump horizontally with rates proportional to the particle number (as for an independent system) and with constant rates in the vertical direction. We will come back to this model later. The question we ask here is more general and is for the microscopic system: how do we understand the presence of long-range spatial correlations in generic perturbations of the independent particle system? All the perturbations we consider make the system anisotropic in the sense that the dynamics no longer possesses the full symmetry of the lattice. It is this ingredient together with the conservation law (of the number of particles) that is responsible for the weak (algebraic) decay of the correlations. The key mechanism by which the system settles down in such a critical state is, as we will see, that the temporal correlations, and this even for independent particles, have a slow decay. If the stationary state does not possess a local Markov property (as in the definition of Gibbs states), then points far away in space must in the expansion be connected with each other via spacetime paths with massless propagator (essentially, the inverse Laplacian). This can be thought of as a non-vanishing correction to the usual high temperature expansion in equilibrium statistical mechanics. Hence, as the stationary state of these processes generically is not expected to be Gibbsian for a local interaction potential, weak decay in space will also be forced upon the system. Summarizing, the long-range spatial correlations are caused by the free spacetime propagator being massless (as a result of the conservation law) but this effect may vanish in special (or non-generic) situations, e.g. in the isotropic case when the stationary state is a Gibbs state.

Note that we do not consider non-equilibrium models with boundary conditions driving the system. There is an extensive literature on the critical properties of these systems including experimental work [10–13]. We concentrate on homogeneous infinite systems and there is no external parameter to tune the model on a critical surface.

Finally a word on our expansions. The process is started from a product state of Poisson measures and we first obtain the expansion for finite times using a Dyson formula. For the equal time correlations and under some extra conditions on the rates including the case of the Van Beijeren perturbation [9], we show the convergence of the perturbation series for small times. We then take the formal $t\to\infty$ limit to obtain information about the associated stationary state. The long-range correlations are found from a term-by-term analysis of the series. This is explicitly carried out up to second order in the expansion parameter for some specific model and we show how imposing the condition of detailed balance, by restoring the isotropy of the model, simplifies the various terms to make the spatial correlations again short range.

The next section introduces the notation and definitions of the model. Section 3 contains the derivation of the perturbation expansion. The discussion of long-range correlations is in section 4.

2. The model

Let $\{e_\alpha\}_{\alpha=1}^d$ be the d unit vectors of $\mathbb{Z}^d$ pointing in the positive α-direction. The dynamics consists of transitions from particle configuration $\eta \in \mathbb{N}^{\mathbb{Z}^d}$ to $\eta^{x,x+e_\alpha}$ where $x \in \mathbb{Z}^d$ and

$$\eta^{x,y} \equiv \eta + \delta_y - \delta_x \tag{2.1}$$

for $\delta_z \equiv$ the configuration with one particle at site $z \in \mathbb{Z}^d$, all other sites empty. The sum and difference in (2.1) are site-wise. The different particles are indistinguishable. The transition rates $c_\alpha(\eta(x))$ are non-negative functions of the particle number $\eta(x) = 0, 1, 2, \ldots$ at $x \in \mathbb{Z}^d$. They can be understood as the probability per unit time that a particle jumps from x to $x + e_\alpha$ or to $x - e_\alpha$, $\alpha = 1, \ldots, d$, and are written as

$$c_\alpha(\eta(x)) \equiv \eta(x) + \beta_\alpha(\eta(x)) \tag{2.2}$$

where $\beta_\alpha(0) = 0$, $\beta_\alpha(n) \geqslant 0$ for $n \in \mathbb{N}$, is the perturbation.

For a finite volume $\Lambda \subset \mathbb{Z}^d$, say with periodic boundary conditions, the master equation for the probability $P_\Lambda(\eta; t)$ to find a certain configuration $\eta \in \mathbb{N}^\Lambda$ at time $t \geqslant 0$ is

$$\begin{aligned}\frac{\partial}{\partial t} P_\Lambda(\eta; t) = \tfrac{1}{2} \sum_{x \in \Lambda} \sum_{\alpha=1}^{d} \{ & c_\alpha(\eta(x+e_\alpha)+1) P_\Lambda(\eta^{x,x+e_\alpha}; t) \\ & + c_\alpha(\eta(x-e_\alpha)+1) P_\Lambda(\eta^{x,x-e_\alpha}; t) - 2c_\alpha(\eta(x)) P_\Lambda(\eta; t)\}. \end{aligned}\tag{2.3}$$

Usually, one starts with a finite number of particles and then constructs the infinite particle dynamics by letting their number diverge (thermodynamic limit). The rates (2.2) must then satisfy some minimal conditions to ensure the existence of this limiting dynamics but we will not go into this here, see e.g. [5] and [14]. For example, it suffices to take $\beta_\alpha(n)$, $n \in \mathbb{N}$ a non-decreasing function with

$$\beta_\alpha(n+1) \leqslant \mathrm{e}^{cn} \beta_\alpha(n)$$

$n > 0$, for some constant c. Mathematically, the particle system is a Markov process η_t, $t \geqslant 0$, defined on a dense subset of $\mathbb{N}^{\mathbb{Z}^d}$ for which the generator L is first defined on local functions $f(\eta)$, $\eta \in \mathbb{N}^{\mathbb{Z}^d}$, by

$$Lf(\eta) \equiv \tfrac{1}{2} \sum_{x \in \mathbb{Z}^d} \sum_{\alpha=1}^{d} c_\alpha(\eta(x))[f(\eta^{x,x+e_\alpha}) + f(\eta^{x,x-e_\alpha}) - 2f(\eta)]. \tag{2.4}$$

We then have that

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathbb{E}^\eta[f(\eta_t)] = \mathbb{E}^\eta[Lf(\eta_t)] \tag{2.5}$$

with $\mathbb{E}^\eta$ the expectation in the process (2.4) started from particle configuration η.

Let

$$\Omega \equiv \left\{ \xi \in \mathbb{N}^{\mathbb{Z}^d} : |\xi| \equiv \sum_{x \in \mathbb{Z}^d} \xi(x) < \infty \right\} \tag{2.6}$$

denote the finite particle configurations. ξ will be used to specify the positions of $|\xi|$ independent simple random walkers. The probability that a collection of such walkers starting from ξ end up in ξ' after time t is denoted by $P_t(\xi, \xi')$. Obviously, $P_t(\xi, \xi') = 0$

unless $|\xi|=|\xi'|$. For example, if ξ, respectively ξ', correspond to configurations with one particle, say at site x and at site y, then $P_t(\xi,\xi')=p_t(x,y)$ and the equations

$$\begin{aligned}&p_0(x,y)=\delta_{x,y}\\&\frac{\partial}{\partial t}p_t(x,y)=\tfrac{1}{2}\sum_{\alpha=1}^{d}[p_t(x,y+e_\alpha)+p_t(x,y+e_\alpha)-2p_t(x,y)]\end{aligned}\tag{2.7}$$

define the transition probabilities.

Since the unperturbed system consists of independent random walkers, it is useful to introduce a set of functions of the particle configuration which transform simply under this free evolution. Define therefore, for $\xi\in\Omega$, $\eta\in\mathbb{N}^{\mathbb{Z}^d}$,

$$D(\xi,\eta)\equiv\prod_{x\in\mathbb{Z}^d}D_{\xi(x)}(\xi(x))\tag{2.8}$$

where

$$\begin{aligned}D_k(n)&=\frac{n!}{(n-k)!}&&\text{if } 0\leqslant k\leqslant n\\&=0&&\text{if } k>n\qquad n\in\mathbb{N}\end{aligned}\tag{2.9}$$

is the Poisson polynomial of order k. The following recursion relations will prove to be useful:

$$\begin{aligned}&\text{(i)}\quad \frac{(-1)^k}{k!}D_k(n-1)=\sum_{j=0}^{k}\frac{(-1)^j}{j!}D_j(n)\\&\text{(ii)}\quad D_k(n+1)=D_k(n)+kD_{k-1}(n)\\&\text{(iii)}\quad nD_k(n-1)=D_{k+1}(n)\qquad nD_k(n)=D_{k+1}(n)+kD_k(n).\end{aligned}\tag{2.10}$$

Consider now the independent particle system with transition rates $c_\alpha(n)=n$, see (2.2), and let L_0 be the corresponding generator, see (2.4). From (2.8) and (2.10) (ii), (iii)) we easily find that (see also p 33 in [5])

$$[L_0D(\xi,\cdot)](\eta)=[L_0D(\cdot,\eta)](\xi)\tag{2.11}$$

that is, the action of L_0 on the function $D(\xi,\eta)$ is the same both when it acts on the ξ and on the η variables. Hence, if $\mathbb{E}_0^\sigma$ is the corresponding expectation for the independent system with initial configuration σ, then combining (2.11) and (2.5) gives

$$\mathbb{E}_0^\sigma D(\xi,\eta_t)=\mathbb{E}_0^\xi D(\xi_t,\eta)\tag{2.12}$$

and the following duality relation holds:

$$\mathbb{E}_0^\eta(\xi,\eta_t)=\sum_{\xi'\in\Omega}P_t(\xi,\xi')D(\xi',\eta).\tag{2.13}$$

From (2.13) it is easy to show that for all $\rho\geqslant 0$, the product measure

$$\nu_\rho\equiv\prod_{x\in\mathbb{Z}^d}\mathscr{P}_\rho^x\tag{2.14}$$

is invariant for the independent particle system, with $\mathscr{P}_\rho^x\equiv$ Poisson measure on $\mathbb{N}$ with parameter ρ having probability distribution

$$\mathscr{P}_\rho^x(n)=\mathrm{e}^{-\rho}\frac{\rho^n}{n!}.\tag{2.15}$$

Moreover, the functions (2.9) span all polynomials $r(n)$, $n \in \mathbb{N}$ in the sense that there are numbers $\lambda(r, k)$, $k \in \mathbb{N}$, such that

$$r(n) = \sum_{k=0}^{\infty} \lambda(r, k) D_k(n)$$

$$\lambda(r, k) = \frac{1}{k!} \frac{d^k}{d\rho^k} \nu_\rho(r)\bigg|_{\rho=0} \tag{2.16}$$

$$\nu_\rho(f) \equiv \mathrm{e}^{-\rho} \sum_{n=0}^{\infty} f(n) \frac{\rho^n}{n!}.$$

Similarly, local functions $f(\eta)$, $\eta \in \mathbb{N}^{\mathbb{Z}^d}$, which do not increase too fast as $|\eta| \to \infty$, can be decomposed as

$$f(\eta) = \sum_{\xi} \lambda(f, \xi) D(\xi, \eta). \tag{2.17}$$

We can thus say that the perturbation

$$L_1 f(\eta) \equiv \tfrac{1}{2} \sum_{x \in \mathbb{Z}^d} \sum_{\alpha=1}^{d} \beta_\alpha(\eta(x))[f(\eta^{x,x+e_\alpha}) + f(\eta^{x,x-e_\alpha}) - 2f(\eta)] \tag{2.18}$$

is completely characterized by the numbers $q(\xi, \xi')$, ξ, $\xi' \in \Omega$, defined by

$$L_1 D(\xi, \eta) = \sum_{\xi'} q(\xi, \xi') D(\xi', \eta). \tag{2.19}$$

Given a specific zero-range process it is of course in general a long but straightforward computation to use relations like (2.10) to find the corresponding $q(\xi, \xi')$. An explicit formula can be obtained in terms of the numbers

$$\{\lambda_\alpha(k, l), k, l \in \mathbb{N}, \alpha = 1, \ldots, d\}$$

for which

$$\beta_\alpha(n) D_k(n) \equiv \sum_{l=0}^{\infty} \lambda_\alpha(k, l) D_l(n). \tag{2.20}$$

If $\beta_\alpha(n)$ is a polynomial of degree a, then $\lambda_\alpha(k, l) = 0$ whenever $l > k + a$. In general,

$$q(\xi, \xi + l\delta_x) = v(\xi(x), \xi(x) + l) \qquad l \neq 0 \qquad l + \xi(x) \geqslant 0$$

$$q(\xi, \xi) = \sum_{x \in \mathbb{Z}^d} v(\xi(x), \xi(x)) \tag{2.21}$$

$$q(\xi, \xi + l\delta_x - \delta_{x \pm e_\alpha}) = w_\alpha(\xi(x), \xi(x \pm e_\alpha), \xi(x) + l), \; l + \xi(x) \geqslant 0$$

and $q(\xi, \xi')$ is zero otherwise. $l\delta_x$, $l \in \mathbb{Z}$, has to be understood as in (2.1)—if l is negative, then $|l|$ number of particles have to be removed—and

$$v(k, l) \equiv k!(-1)^k \sum_{j=0}^{k-1} \frac{(-1)^j}{j!} \sum_{\alpha=1}^{d} \lambda_\alpha(j, l) \qquad k > 0$$

$$v(0, l) \equiv 0 \tag{2.22}$$

$$w_\alpha(k, k', l) \equiv \tfrac{1}{2} k' k!(-1)^k \sum_{j=0}^{k} \frac{(-1)^j}{j!} \lambda_\alpha(j, l).$$

As examples, if $\beta_\alpha(n)=\gamma$ for $n>0$ and $\alpha=1$, $\beta_\alpha(n)=0$ otherwise, then

$$\begin{aligned} v(k,l) &= \gamma(-1)^{k+l+1}\frac{k!}{l!} && l>k\neq 0 \\ &= 0 && \text{otherwise} \end{aligned} \tag{2.23}$$

and

$$\begin{aligned} w_\alpha(k,k',l) &= \frac{\gamma}{2}k'(-1)^{k+l+1}\frac{k!}{l!} && l>k \qquad \alpha=1 \\ &= 0 && \text{otherwise.} \end{aligned} \tag{2.24}$$

If $\beta_\alpha(n)=n^2$ for $\alpha=1$ and zero otherwise, then

$$\begin{aligned} v(k,l) &= -k^2 && k=l \\ &= -k && k=l-1 \\ &= 0 && \text{otherwise} \end{aligned} \tag{2.25}$$

and

$$\begin{aligned} w_\alpha(k,k',l) &= \tfrac{1}{2}k' && k=l-2 \qquad \alpha=1 \\ &= \frac{(k+1)}{2}k' && k=l-1 \qquad \alpha=1 \\ &= 0 && \text{otherwise.} \end{aligned} \tag{2.26}$$

Obviously, there are many models for which we know the stationary states. The easiest way to obtain them is by requiring that the condition of detailed balance be satisfied. Formally, from (2.3), if

$$c_\alpha(\eta(x))P(\eta)=c_\alpha(\eta(x\pm e_\alpha)+1)P(\eta^{x,x\pm e_\alpha}) \tag{2.27}$$

for all $\eta\in\mathbb{N}^{\mathbb{Z}^d}$, $x\in\mathbb{Z}^d$, $\alpha=1,\ldots,d$, then $P(\eta)$ is a reversible measure for the process (2.4). If, therefore, we require that for some constants γ_α

$$c_\alpha(n)=\gamma_\alpha c(n) \qquad \alpha=1,\ldots,d \tag{2.28}$$

and define the probability measures μ_ρ^x on $\mathbb{N}$ by

$$\mu_\rho^x(\eta(x)=n)\equiv Z_\rho^{-1}\frac{\rho^n}{c(1)\ldots c(n)} \tag{2.29}$$

where ρ is such that the normalization factor z_ρ is finite, then the Gibbs measures

$$\mu_\rho\equiv\prod_{x\in\mathbb{Z}^d}\mu_\rho^x \tag{2.30}$$

are invariant and reversible. It is therefore essential to allow for such anisotropic dynamics so that (2.28) is not satisfied, to study the consequences of generic perturbations of the independent particle system.

3. The expansion

In the perturbed process, for any $\xi \in \Omega$, $\eta \in \mathbb{N}^{\mathbb{Z}^d}$,

$$\begin{aligned}\mathbb{E}^\eta[D(\xi, \eta_t)] &= \sum_{\xi'} P_t(\xi, \xi')D(\xi', \eta) + \int_0^t \mathrm{d}s \sum_{\xi'} P_{t-s}(\xi, \xi')\mathbb{E}^\eta[L_1 D(\xi', \eta_s)] \\ &= \sum_{\xi'} P_t(\xi, \xi')D(\xi', \eta) + \int_0^t \mathrm{d}s \sum_{\xi', \xi''} P_{t-s}(\xi, \xi')q(\xi', \xi'')\mathbb{E}^\eta[D(\xi'', \eta_s)]. \qquad (3.1)\end{aligned}$$

If we start the process from the Poisson measure ν_ρ, see (2.14), (2.15), then, using

$$\nu_\rho(D(\xi, \cdot)) = \rho^{|\xi|} \qquad (3.2)$$

$$\mathbb{E}^\rho[D(\xi, \eta_t)] = \rho^{|\xi|} + \sum_{\xi', \xi''} \int_0^t \mathrm{d}s\, P_{t-s}(\xi, \xi')q(\xi', \xi'')\mathbb{E}^\rho[D(\xi'', \eta_s)] \qquad (3.3)$$

where

$$\mathbb{E}^\rho[\cdot] \equiv \nu_\rho[\mathbb{E}^\eta[\cdot]].$$

Upon iterating (3.3), we formally obtain

$$\mathbb{E}^\rho[D(\xi, \eta_t)] = \sum_{l=0}^{\infty} V_\xi^{(l)}(t) \qquad (3.4)$$

where

$$\begin{aligned} V_\xi^{(0)}(t) &\equiv \rho^{|\xi|} \\ V_\xi^{(l)}(t) &\equiv \sum_{\xi'} \int_0^t \mathrm{d}s\, P_{t-s}(\xi, \xi')r_{\xi'}^{(l)}(s) \qquad (3.5) \\ r_\xi^{(l)}(t) &\equiv \sum_{\xi'} q(\xi, \xi')\, V_{\xi'}^{(l-1)}(t) \qquad l > 0. \end{aligned}$$

If the perturbation L_1 (or, equivalently, β_α) is of order γ, then $V_\xi^{(l)}(t)$ is of order γ^l and (3.4) represents an expansion around $\gamma = 0$.

Of course, it is wrong to say that L_1 (even for small but fixed γ) is always a 'small' perturbation of L_0 (e.g. in the sense of Dirichlet forms) since the rates $\beta_\alpha(n)$ may very well dominate in $c_\alpha(n)$. This happens for example in (2.25), (2.26). Still, for fixed n, $c_\alpha(n) \sim n$ as $\gamma \downarrow 0$.

The physical interpretation of (3.5) consists of two parts: at time s a collision or scattering occurs between $|\xi'|$ particles to produce $|\xi|$ new particles with amplitude governed by $q(\xi, \xi')$; these then evolve freely for a time $t - s$ until the next collision occurs. The index l in (3.4) thus counts the number of such collisions that have happened since the particles started. Letting $\gamma \to 0$ (low density of collision points) corresponds therefore to some kind of a kinetic limit.

The first question is to see when (3.4), (3.5) indeed define a convergent expansion for the equal time correlation functions of the corresponding zero-range process. We have two types of results.

Proposition 3.1. Suppose there is a function $f(n)$, $n \in \mathbb{N}$, with $s(a) \equiv \nu_a(f) < \infty$, for all $a > 0$, such that for all n, $R \geq 0$ and for some $\gamma < \infty$,

$$\sup_{|\xi| = n} \sum_{|\xi'| = n \pm R} |q(\xi, \xi')| \leq \gamma \frac{n}{R!} f(R). \qquad (3.6)$$

Then, (3.4), (3.5) define a convergent expansion for $\mathbb{E}^\rho[D(\xi, \eta_t)]$ provided that the time t is sufficiently small (see further in (3.15) to see *how* small). Furthermore, for fixed t, $|V_\xi^{(l)}(t)| \leqslant \gamma^l$ as $\gamma \downarrow 0$.

Remarks on proposition 1

1. Condition (3.6) is satisfied for the van Beijeren perturbation [9] as defined in (2.23), (2.24) with $f(n) \equiv 1$. More generally, we expect that (3.6) holds for every perturbation with the $\{\beta_\alpha(n)\}_\alpha$ uniformly bounded in n.

2. While the collision kernel $q(\xi, \xi')$ can have small amplitude γ, it is still in many cases an unbounded function of ξ (or ξ'). The assumption (3.6) does not permit this in general. However, as we will see in the proof, (3.6) can be replaced by the weaker (but more complicated) condition that for all functions F on Ω, $l \geqslant 1$, $s \geqslant 0$

$$\sup_{|\xi|=n} \left| \int_0^s \mathrm{d}\tau \sum_{\xi'} \mathbb{E}^\xi[q(\xi_{s-\tau}, \xi')F(\xi')\tau^{l-1}\right| \leqslant n\gamma \frac{s^l}{l} \sum_{R=0}^{\infty} \sup_{|\xi'|=n+R} |F(\xi')| \frac{f(R)}{R!}. \tag{3.7}$$

This time-integrated form of (3.6) will in some cases be useful to overcome the unboundedness of $q(\xi, \xi')$ as it is the case that with large probability (at least in sufficiently high dimensions) $\xi_{s-\tau}(x) \leqslant 1$, $\forall x \in \mathbb{Z}^d$, $s-\tau$ large, even if all the random walkers were initially concentrated at one site.

Proof of proposition 1. From the hypothesis (3.6) (only $\rho > 1$ to be considered),

$$\begin{aligned} |V_\xi^{(1)}(t)| &\leqslant t\gamma|\xi| \sum_{R=0}^{\infty} \rho^{|\xi|+R} \frac{f(R)}{R!} \\ &\leqslant t\gamma|\xi|\rho^{|\xi|} \mathrm{e}^\rho s(\rho). \end{aligned} \tag{3.8}$$

If we assume inductively that for some $l \geqslant 1$,

$$\begin{aligned} |V_\xi^{(l)}(t)| \leqslant \frac{t^l}{l!} \gamma^l \rho^{|\xi|} \mathrm{e}^\rho s(\rho) &\sum_{R_1,\dots,R_{l-1}} |\xi|(|\xi|+R_1)\dots \\ &\times(|\xi|+r_1+\dots R_{l-1}) \frac{f(R_1)}{R_1!} \dots \frac{f(R_{l-1})}{R_{l-1}!} \rho^{R_1+\dots+R_{l-1}} \end{aligned} \tag{3.9}$$

then, again by (3.6) the same bound also holds for $l+1$ replacing l in (3.9).

Let $X_1, \dots, X_i, \dots$ be independently and identically distributed random variables with distribution

$$w(X_i = n) = \frac{f(n)}{n!} \rho^n \frac{\mathrm{e}^{-\rho}}{s(\rho)} \qquad n \in \mathbb{N}. \tag{3.10}$$

Then (3.9) can be bounded by

$$\frac{t^l}{l!} \gamma^l \rho^{|\xi|} [\mathrm{e}^\rho s(\rho)]^l W[(|\xi| + X_1 + \dots + X_{l-1})^l] \tag{3.11}$$

where $W[\cdot]$ denotes the expectation with respect to the distribution (3.10). Using Cramer's theory of large deviations and Varadhan's formula [15], it is easy to show that

$$\lim_{l\uparrow\infty} \frac{1}{l} \log W\left[\left(\frac{|\xi| + X_1 + \dots + X_{l-1}}{l}\right)^l\right] = \sup_{\theta>0} [\log \theta - I(\theta)] \equiv b < \infty \tag{3.12}$$

with entropy function

$$I(\theta) \equiv \sup_{a>0} \left[\theta \log \frac{a}{\rho} - a + \rho - \log \frac{s(a)}{s(\rho)} \right]. \tag{3.13}$$

From Stirling's formula, we thus obtain that for large l,

$$|V_\xi^{(l)}(t)| \leq \rho^{|\xi|} (\gamma t \, \mathrm{e}^{\rho+1} s(\rho) b)^l \tag{3.14}$$

and it suffices to take

$$t < (\gamma \, \mathrm{e}^{\rho+1} s(\rho) b)^{-1} \tag{3.15}$$

in (3.4) to obtain convergence. □

A global result in time is available for bounded perturbations. For example, if in the Van Beijeren perturbation, the rate functions are changed for only a finite number of sites (rendering the system inhomogeneous), then the following holds.

Proposition 3.2. Under the same assumptions as proposition 1 with the RHS of (3.6) replaced by the bound $\gamma[f(R)/R!]$, the expansion (3.4) converges for all times $t < \infty$.

Proof of proposition 2. Immediate from the proof of proposition 1. We find that

$$|V_\xi^{(l)}(t)| \leq \frac{t^l}{l!} \xi^{|\xi|} [\gamma \, \mathrm{e}^\rho s(\rho)]^l. \tag{3.16}$$

If we take the formal $t \to \infty$ limit in (3.4), (3.5), we obtain the expansion

$$\langle D(\xi, \eta) \rangle_\infty = \sum_{l=0}^{\infty} V_\xi^{(l)} \tag{3.17}$$

for the correlation functions in the homogeneous stationary state corresponding to density $\langle \eta(x) \rangle_\infty = \rho$, which, we assume, is reached asymptotically if the process was started from the Poisson measure ν_ρ. This makes sense only if indeed there is a unique invariant measure for this fixed ρ.

In (3.18),

$$\begin{aligned} V_\xi^{(0)} &\equiv \rho^{|\xi|} \\ V_\xi^{(l)} &\equiv \int_0^\infty \sum_{\xi'} \mathbb{E}^\xi [q(\xi_t, \xi')] V_{\xi'}^{(l-1)} \qquad l > 0. \end{aligned} \tag{3.18}$$

To make this formal expansion more explicit, we rewrite (3.17) via (2.21), (2.22). It then becomes

$$\begin{aligned} V_\xi^{(l)} = \tfrac{1}{2} \int_0^\infty \mathrm{d}t \sum_{k=0}^{\infty} \mathbb{E}^\xi \Bigg[\sum_{x,\alpha} \xi_t(x) \{ & K_\alpha(\xi_t(x+e_\alpha)+1, k) V^{(l-1)}_{r^{+\alpha}_{x,k}(\xi_t)} \\ & + K_\alpha(\xi_t(x-e_\alpha)+1, k) V^{(l-1)}_{r^{-\alpha}_{x,k}(\xi_t)} - 2K_\alpha(\xi_t(x), k) V^{(l+1)}_{r^{0}_{x,k}(\xi_t)} \} \Bigg] \qquad l > 0 \end{aligned} \tag{3.19}$$

where

$$K_\alpha(m, n) \equiv (+1)^{m-1} (m-1)! \sum_{j=0}^{m-1} \frac{(-1)^j}{j!} \lambda_\alpha(j, n) \tag{3.20}$$

and

$$r^{\pm\alpha}_{x,k}(\xi) \equiv \xi - \delta_x - \xi(x \pm e_\alpha)\delta_{x\pm e_\alpha} + k\delta_{x\pm e_\alpha}$$
$$r^{0}_{x,k}(\xi) \equiv \xi - \xi(x)\delta_x + k\delta_x \tag{3.21}$$

are new random walk configurations obtained from ξ by changing the number of particles at $x \pm e_\alpha$ and x.

Notice that if $\beta_\alpha(n) = n$, then from (3.20) $K_\alpha(m, n) \sim \delta_{m,n}$ the unit matrix, and substitution in (3.18) yields

$$V^{(l)}_\xi \sim \tfrac{1}{2}\int_0^\infty \mathrm{d}t\, \mathbb{E}^\xi[L_0 V^{(l-1)}_{\xi_t}] = V^{(l-1)}_\xi \qquad l > 0$$

and thus $V^{(l)}_\xi = V^{(1)}_\xi = 0$, which is consistent with the discussion (2.11)-(2.15).

4. Long-range correlations

We begin by investigating the stationary two-points function $\langle \eta(a)\eta(b)\rangle_\infty$, $a, b \in \mathbb{Z}^d$, for a general perturbation up to first order in the expansion (3.16). From (3.19) we get

$$\begin{aligned}
V^{(1)}_{\delta_a+\delta_b} = \tfrac{1}{2}\int_0^\infty \mathrm{d}t \sum_{k=0}^{\infty} \mathbb{E}^{\delta_a+\delta_b} & \\
\times \Bigg[\sum_{x,\alpha} \xi_t(x)\{ & K_\alpha(\xi_t(x+e_\alpha)+1, k)\rho^{1-\xi_t(x+e_\alpha)+k} \\
& + K_\alpha(\xi_t(x-e_\alpha)+1, k)\rho^{1-\xi_t(x-e_\alpha)+k} + 2K_\alpha(\xi_t(x), k)\rho^{2-\xi_t(x)+k}\}\Bigg].
\end{aligned} \tag{4.1}$$

An easy computation gives

$$\begin{aligned}
\mathbb{E}^{\delta_a+\delta_b}&[\xi_t(x)K_\alpha(\xi_t(x \pm e_\alpha)+1, k)\rho^{1-\xi_t(x\pm e_\alpha)+k}] \\
&= (p_t(a, x) + p_t(b, x))K_\alpha(1, k)\rho^{1+k} + (p_t(a, x)p_t(b, x \pm e_\alpha) \\
&\quad + p_t(a, x \pm e_\alpha)p_t(b, x))(K_\alpha(2, k)\rho^k - K_\alpha(1, k)\rho^{1+k})
\end{aligned} \tag{4.2}$$

and

$$\begin{aligned}
\mathbb{E}^{\delta_a+\delta_b}&[\xi_t(x)K_\alpha(\xi_t(x), k)\rho^{2-\xi_t(x)+k}] \\
&= (p_t(a, x) + p_t(b, x))K_\alpha(1, k)\rho^{1+k} + 2p_t(a, x)p_t(b, x)) \\
&\quad \times (K_\alpha(2, k)\rho^k - K_\alpha(1, k)\rho^{1+k})
\end{aligned} \tag{4.3}$$

where $p_t(x, y)$ was introduced in (2.7). Combining (4.1)-(4.3) we obtain

$$\begin{aligned}
V^{(1)}_{\delta_a+\delta_b} = \tfrac{1}{2}\sum_{\alpha=1}^{d}\sum_{k=0}^{\infty} & [K_\alpha(2, k)\rho^k - K_\alpha(1, k)\rho^{1+k}] \\
& \times \int_0^\infty \mathrm{d}t \sum_{x\in\mathbb{Z}^d} \{p_t(a, x)\Delta_\alpha p_t(b, x) + \Delta_\alpha p_t(a, x)p_t(b, x)\}
\end{aligned} \tag{4.4}$$

with $\Delta_\alpha g(x) \equiv g(x+e_\alpha)+g(x-e_\alpha)-2g(x)$, $x \in \mathbb{Z}^d$. Hence

$$V^{(1)}_{\delta_a+\delta_b} = \sum_{\alpha=1}^{d} \chi_\alpha(\rho) \int_0^\infty \mathrm{d}t[p_t(0, b-a+e_\alpha)+p_t(0, b-a+e_\alpha)-2p_t(0, b-a)]$$

where

$$\begin{aligned}\chi_\alpha(\rho) &\equiv \tfrac{1}{2} \sum_{k=0}^{\infty} (K_\alpha(2, k)\rho^k - K_\alpha(1, k)\rho^{1+k} \\ &= \tfrac{1}{2} \sum_{k=0}^{\infty} [\lambda_\alpha(1, k) - \lambda_\alpha(0, k)(1+\rho)]\rho^k \\ &= \tfrac{1}{2} \nu_\rho[c_\alpha(n)(n-1-\rho)] \end{aligned} \tag{4.5}$$

is the expectation value in the Poisson measure (2.14) of the perturbed rate function $c_\alpha(n)$ multiplied by $n-(1+\rho)$ so that $\chi_\alpha(\rho)=0$ whenever $c_\alpha(n) \sim n$.

We conclude therefore that the first-order two-point function can be written simply as

$$V^{(1)}_{\delta_a+\delta_b} = \frac{1}{(2\pi)^d} \int_{(-\pi,\pi]^d} \mathrm{d}p\, \mathrm{e}^{\mathrm{i}p\cdot(b-a)} \frac{\sum_{\alpha=1}^{d} \chi_\alpha(\rho)(1-\cos p\cdot e_\alpha)}{\sum_{\alpha=1}^{d} (1-\cos p\cdot e_\alpha)}. \tag{4.6}$$

Note that if the process is *isotropic* (in the sense of (2.28), then $V^{(1)}_{\delta_a+\delta_b}=0$ whenever $a \neq b$. This is consistent with the discussion at the end of section 2. In general, however, the structure function (to first order)

$$\hat{S}(p) = \frac{\sum_\alpha \chi_\alpha(\rho)(1-\cos p\cdot e_\alpha)}{\sum_\alpha (1-\cos p\cdot e_\alpha)} \tag{4.7}$$

is *not* analytic around $p=0$. That is, if $\chi_\alpha(\rho) \neq \chi_{\alpha'}(\rho)$ for some α, $\alpha'=1,\ldots,d$, then $\lim_{p\to 0} \hat{S}(p)$ depends on the way of approach $p \to 0$. This cannot happen for isotropic dynamics but for generic perturbations, formula (4.7) describes the long-distance behaviour ($p \to 0$) of a quadrupole field. In real space, to first order in the expansion, it implies a decay

$$\langle \eta(0)\eta(x)\rangle_\infty - \rho^2 \simeq \sum_{\alpha=1}^{d} \left[\chi_\alpha(\rho) - \frac{1}{d}\sum_{\alpha'} \chi_{\alpha'}(\rho)\right] \frac{x^2}{|x|^{d+2}} \tag{4.8}$$

as $|x| \to \infty$. These are the long-range correlations we mention in the title and they are of the same form as found in [8]. In particular, this phenomenon is realized for the examples (2.23)-(2.26). For the model treated in [9], (2.23), (2.24),

$$\begin{aligned}\chi_\alpha(\rho) &= \frac{\gamma}{2}[\mathrm{e}^{-\rho}(1+\rho)-1] \qquad &&\text{if } \alpha=1 \\ &= 0 &&\text{otherwise.}\end{aligned} \tag{4.9}$$

For (2.25), (2.26)

$$\begin{aligned}\chi_\alpha(\rho) &= \tfrac{1}{2}\rho^2 \qquad &&\text{if } \alpha=1 \\ &= 0 &&\text{otherwise.}\end{aligned} \tag{4.10}$$

For the three-points function $\langle\eta(a)\eta(b)\eta(c)\rangle_\infty$ to first order, we again start from (3.19) to obtain

$$V^{(1)}_{\delta_a+\delta_b+\delta_c}=\tfrac{1}{2}\sum_{\alpha=1}^{d}\sum_{k=0}^{\infty}[K_\alpha(2,k)\rho^{1+k}-K_\alpha(1,k)\rho^{2+k}]$$
$$\times\int_0^\infty \mathrm{d}t\sum_{x\in\mathbf{Z}^d}\{p_t(a,x)\Delta_\alpha p_t(b,x)+\Delta_\alpha p_t(a,x)p_t(b,x)$$
$$+p_t(a,x)\Delta_\alpha p_t(c,x)+\Delta_\alpha p_t(a,x)p_t(c,x)$$
$$+p_t(b,x)\Delta_\alpha p_t(c,x)+\Delta_\alpha p_t(b,x)p_t(c,x)\}$$
$$+\tfrac{1}{2}\sum_{\alpha=1}\sum_{k=0}^{\infty}[K_\alpha(3,k)\rho^{k}-2K_\alpha(2,k)\rho^{1+k}+K_\alpha(1,k)\rho^{2+k}]$$
$$\times\int_0^\infty\sum_{x\in\mathbf{Z}^d}\{p_t(a,x)p_t(b,x)\Delta_\alpha p_t(c,x)$$
$$+p_t(a,x)\Delta_\alpha p_t(b,x)p_t(c,x)+\Delta_\alpha p_t(a,x)p_t(b,x)p_t(c,x)\}. \tag{4.11}$$

Thus, taking Fourier transforms, we end up with

$$V^{(1)}_{\delta_a+\delta_b+\delta_c}=\rho[V^{(1)}_{\delta_a+\delta_b}+V^{(1)}_{\delta_a+\delta_c}+V^{(1)}_{\delta_b+\delta_c}]$$
$$+\frac{1}{(2\pi)^{2d}}\int_{(-\pi,\pi]^{2d}}\mathrm{d}p\int\mathrm{d}p'\,\mathrm{e}^{\mathrm{i}p\cdot(b-a)}\,\mathrm{e}^{\mathrm{i}p'\cdot(c-a)}$$
$$\times\frac{\sum_{\alpha=1}^{d}\tilde{\chi}_\alpha(\rho)[3-\cos p\cdot e_\alpha+\cos p'\cdot e_\alpha-\cos(p+p')\cdot e_\alpha]}{\sum_{\alpha=1}^{d}[3-\cos p\cdot e_\alpha-\cos p'\cdot e_\alpha-\cos(p+p')\cdot e_\alpha]} \tag{4.12}$$

and similar expressions can be obtained for still higher correlation functions. In (4.12)

$$\tilde{\chi}_\alpha(\rho)\equiv\tfrac{1}{2}\sum_{k=0}^{\infty}[K_\alpha(3,k)+2K_\alpha(2,k)\rho+K_\alpha(1,k)\rho^2]\rho^k$$
$$=\tfrac{1}{2}\,\nu_\rho(c_\alpha(n)n^2)-(3+2\rho)\nu_\rho(c_\alpha(n)n)+(2+2\rho+\rho^2)\nu_\rho(c_\alpha(n)) \tag{4.13}$$

which is zero if $c_\alpha(n)\sim n$. We thus get in general that to first order the expectations of functions with support separated a distance r in space, in dimension $d\geqslant 2$, have an algebraic decay with magnitude proportional to r^{-d} as $r\to\infty$.

To study the two-points functions $\langle\eta(a)\eta(b)\rangle_\infty$ also to higher orders in the expansion, we must investigate terms of the form (3.19) with $\xi=\delta_a+\delta_b$. After taking the expectation with respect to the random walk process ξ_t, we get the generalization of (4.4),

$$V^{(l)}_{\delta_a+\delta_b}=\tfrac{1}{2}\sum_{\alpha=1}^{d}\sum_{k=0}^{\infty}[K_\alpha(2,k)V^{(l-1)}_{k\delta_0}-K_\alpha(1,k)V^{(l-1)}_{(k+1)\delta_0}]$$
$$\times\int_0^\infty\mathrm{d}t\sum_{x\in\mathbf{Z}^d}[\Delta_\alpha p_t(a,x)p_t(b,x)+p_t(a,x)\Delta_\alpha p_t(b,x)]$$
$$+\tfrac{1}{2}\sum_{\alpha=1}^{d}\sum_{k=0}^{\infty}K_\alpha(1,k)\int_0^\infty\mathrm{d}t\sum_{x,y\in\mathbf{Z}^d}[\Delta_\alpha p_t(a,x)p_t(b,y)$$
$$+p_t(a,y)\Delta_\alpha p_t(b,x)]\,V^{(l-1)}_{\delta_y+k\delta_x}\qquad, l\geqslant 1. \tag{4.14}$$

The first term in (4.14) is exactly of the same form as $V^{(1)}_{\delta_a+\delta_b}$, see (4.4), and, for the same reason, implies the decay (4.8) with suitable coefficients. For the second term, which vanishes if $l=1$, we need information on the higher correlation functions to order $l-1$ in the expansion. To simplify matters, we take $l=2$, the second-order term, and we choose the example (2.25), (2.26). Since in this case $K_\alpha(1,k)=1$ if $k=1$, 2, $\alpha=1$ and is zero otherwise, we only have to worry about $V^{(1)}_{\delta_x+\delta_y}$ and $V^{(1)}_{2\delta_x+\delta_y}$. Both have been computed explicitly with results (4.6)-(4.12). The term $k=1$ in the second part of (4.14) is then

$$\frac{1}{2}\int_0^\infty \mathrm{d}t \sum_{x,y} [\Delta_1 p_t(a,x)p_t(b,y)+p_t(a,y)\Delta_1 p_t(b,x)]\, V^{(1)}_{\delta_x+\delta_y}$$

$$=\frac{1}{2}\rho^2\frac{1}{(2\pi)^d}\int_{(-\pi,\pi]^d}\mathrm{d}p\, \mathrm{e}^{\mathrm{i}p\cdot(b-a)}\left[\frac{1-\cos p\cdot e_1}{\sum_{\alpha=1}^d(1-\cos p\cdot e_\alpha)}\right]^2 \tag{4.15}$$

where we have substituted in (4.6) the coefficients (4.10). Similarly for the $k=2$ term, we must substitute (4.12) with, for our example, $\tilde{\chi}_\alpha(\rho)=\frac{1}{2}\rho^2$, if $\alpha=1$ and zero otherwise. This term then becomes

$$\rho^3\frac{1}{(2\pi)^d}\int_{(+\pi,\pi]^d}\mathrm{d}p\,\mathrm{e}^{\mathrm{i}p\cdot(b-a)}\left[\frac{1-\cos p\cdot e_1}{\sum_{\alpha=1}^d(1-\cos p\cdot e_\alpha)}\right]^2$$

$$+\rho^2\frac{1}{(2\pi)^d}\int_{(-\pi,\pi]^d}\mathrm{d}p\,\mathrm{e}^{p\cdot(b-a)}\frac{1-\cos p\cdot e_1}{\sum_{\alpha=1}^d(1+\cos p\cdot e_\alpha)}R(p) \tag{4.16}$$

where

$$R(p)\equiv\frac{1}{2}\frac{1}{(2\pi)^d}\int_{(-\pi,\pi]^d}\mathrm{d}p'\frac{3+\cos p\cdot e_1-\cos p'\cdot e_1-\cos(p+p')\cdot e_1}{\sum_{\alpha=1}^d[3-\cos p\cdot e_\alpha-\cos p'\cdot e_\alpha-\cos(p+p')\cdot e_\alpha]}. \tag{4.17}$$

Bringing all these terms together in (4.14) we conclude that the weak decay of the two-point function also persists to second order in the expansion. Again, the corresponding structure function is homogeneous in the Fourier vector p. The long-distance behaviour is, as in (4.8), of quadrupole type. The same idea applies to all perturbations where $\beta_\alpha(n)$ is a polynomial of degree at most two.

For more general perturbations we have to go back to (4.14) from which we derive that for all $l\geqslant 1$,

$$V^{(l)}_{\delta_a+\delta_b}=\frac{1}{(2\pi)^d}\int_{(-\pi,\pi]^d}\mathrm{d}p\,\mathrm{e}^{\mathrm{i}p\cdot(b+a)}\frac{\sum_{\alpha=1}^d[\chi_\alpha^{(l)}(\rho)+S_\alpha^{(l)}(p)](1-\cos p\cdot e_\alpha)}{\sum_{\alpha=1}^d(1-\cos p\cdot e_\alpha)} \tag{4.18}$$

where

$$\chi_\alpha^{(l)}(\rho)\equiv\sum_{k=0}^\infty[K_\alpha(2,k)V^{(l-1)}_{k\delta_0}-K_\alpha(1,k)V^{(l-1)}_{(k+1)\delta_0}] \tag{4.19}$$

and

$$S_\alpha^{(l)}(p)\equiv\sum_{k=0}^\infty K_\alpha(1,k)\sum_z \mathrm{e}^{\mathrm{i}p\cdot z}[V^{(l-1)}_{\delta_0+k\delta_z}-\rho V^{(l-1)}_{k\delta_0}]. \tag{4.20}$$

Again we see that, in general, generic anisotropy causes the two-point function to decay algebraically (as in (4.8)) to every order in the expansion.

So far we have treated translation invariant systems. Adding a finite perturbation breaks this homogeneity but it is interesting that the effect of long-range correlations persists. We demonstrate this by making a first-order calculation for the perturbation

$$L_1 f(\eta) \equiv \frac{\gamma}{2}[\eta(0)]^2[f(\eta^{0,e_1}) + f(\eta^{0,-e_1}) - 2f(\eta)]. \tag{4.21}$$

Particles jump independently except if they are at the origin and want to jump in the direction e_1. (4.1) now needs to be replaced by

$$\begin{aligned} V^{(1)}_{\delta_a+\delta_b} &= \frac{\gamma}{2}\rho^3 \int_0^\infty \mathrm{d}t\, \mathbb{E}^{\delta_a+\delta_b}[\xi_t(e_1) + \xi_t(-e_1) - 2\xi_t(0)] \\ &\quad + \frac{\gamma}{2}\rho^2 \int_0^\infty \mathrm{d}t\, \mathbb{E}^{\delta_a+\delta_b}[\xi_t(e_1)(\xi_t(0)+1) + \xi_t(-e_1)(\xi_t(0)+1) - 2(\xi_t(0))^2]. \end{aligned}$$

On the other hand,

$$V^{(1)}_{\delta_z} = \frac{\gamma}{2}(\rho + \rho^2) \int_0^\infty \Delta_1 p_t(0, x)\, \mathrm{d}t \tag{4.22}$$

so that we find that in this case, for $a \neq b$,

$$\begin{aligned} \langle \eta(a); \eta(b) \rangle^{(1)} &\\ &\approx \gamma \frac{\rho^2}{2} \int_0^\infty \mathrm{d}t \{ p_t(0, a)[p_t(e_1, b) + p_t(-e_1, b) - 2p_t(0, b)] \\ &\quad + [p_t(e_1, a) + p_t(-e_1, a) - 2p_t(0, a)] p_t(0, b) \} \\ &= \frac{\gamma}{(2\pi)^{2d}} \rho^2 \int_{(-\pi,\pi]^{2d}} \mathrm{d}p \int \mathrm{d}p'\, \mathrm{e}^{\mathrm{i}p\cdot a}\, \mathrm{e}^{\mathrm{i}p'\cdot b} \\ &\quad \times \frac{2 - \cos p\cdot e_1 - \cos p'\cdot e_1}{\Sigma_{\alpha=1}^d [2 - \cos p\cdot e_\alpha - \cos p'\cdot e_\alpha]}. \end{aligned}$$

As a consequence, to first order $\langle \eta(a)\eta(b)\rangle_\infty + \langle \eta(a)\rangle_\infty \langle \eta(b)\rangle_\infty$ decays like r^{-2d}, $d \geqslant 2$, as $|a-b| \equiv r \to \infty$. We thus get a different (faster) decay than for the translation invariant situation but it is still long range. The same behaviour also persists to second order and for other perturbations as those discussed in section 3, proposition 2, but we do not give the explicit computation here. The effect of replacing d by $2d$ as 'effective' dimension in dynamics where a finite perturbation around the origin is added, can also be observed in [16] where a sink/source is added to the simple exclusion process. The authors find a decay for the two-point function as $r^{-(2d-2)}$, $d \geqslant 3$, to be compared with the $r^{-(d-2)}$ of [17] where a density gradient is imposed in the system via the boundary conditions.

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